

Code: EC3T2

II B.Tech - I Semester–Regular/Supplementary Examinations
November 2017

PROBABILITY THEORY AND STOCHASTIC PROCESS
(ELECTRONICS & COMMUNICATION ENGINEERING)

Duration: 3 hours

Max. Marks: 70

PART – A

Answer *all* the questions. All questions carry equal marks
11x 2 = 22 M

1.

a) Explain the following in brief:

- i) Probability as a relative frequency
- ii) Conditional probability

b) Two boxes are selected Randomly. The first box contains 2 white balls and 3 black balls. The second box contains 3 white and 4 black balls. What is the probability of drawing a white ball?

c) Define Cumulative Distribution Function and State Its properties.

d) If the probability density of a random variable is given by

$$f(x) = \begin{cases} c \exp(-x/4) & 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the value that C must have and evaluate $F_x(0.5)$

- e) Explain the Characteristic function of a random variable in detail.
- f) State the Properties of Joint Distribution Function.
- g) Explain the Wide sense stationary & Ergodic random processes.
- h) Given the auto correlation function for a stationary ergodic process with no periodic components is

$$R_{xx}(\tau) = 25 + \frac{4}{1 + 6\tau^2}.$$

Find mean and variance of process X(t).

- i) What is cross power density spectrum? State its properties.
- j) A random process n(t) has a power spectral density $G(f) = n/2$ for $-\infty \leq f \leq \infty$. The random process is passed through a low pass filter which has trans function

$$H(f) = 2 \quad \text{for} \quad -f_m \leq f \leq f_m$$

$$= 0 \quad \text{otherwise}$$

Find the PSD of the waveform at the output of the filter.

- k) Write the Properties of band limited Random processes.

PART – B

Answer any **THREE** questions. All questions carry equal marks.
3 x 16 = 48 M

2. a) State and Prove the Baye's Theorem. 6 M

b) Letter is known to have come either from LONDON or

CLIFTON. On the post card only. Two consecutive letters as 'ON' are legible what is the chance that it came from LONDON. 6 M

c) A pack contains 4 white and 2 green pencils, another contains 3 white and 5 green pencils. If one pencil is drawn from each pack find the probability that

i) Both are white and 2 M

ii) One is white and another is green. 2 M

3. a) State and prove Chebychev's inequality. 8 M

b) A random variable X is uniformly distributed on the interval $(-\pi/2, \pi/2)$. X is transformed to the new random variable $Y = T(X) = a \tan(X)$, where $a > 0$. Find the probability density function of Y. 8 M

4. a) Show that a linear transformation of Gaussian Random variables produces Gaussian random variables. 8 M

b) Random variables X and Y have the joint density function

$$f_{xy}(x, y) = \frac{(x + y)^2}{40} \quad -1 < X < 1 \text{ and } -3 < Y < 3 .$$

i) Find all the second order moments for X and Y. 4 M

ii) σ_x^2 & σ_y^2 2 M

iii) ρ 2 M

5. a) State and prove Wiener-Khinchin relations. 8 M
- b) Consider a random process $X(t) = \cos(\omega t + \theta)$ where 'w' is real constant and θ is a uniform variable in $(0, \pi/2)$. Show that $X(t)$ is not a WSS process. Also find the average power in the process. 8 M
6. a) Define the following random processes:
- i) Band pass 3 M
 - ii) Band limited 3 M
 - iii) Narrow band and its Properties 6 M
- b) A random process $X(t)$ is applied to a network with impulse response $h(t) = u(t) \exp(-bt)$ where $b > 0$ is constant. The cross correlation of $X(t)$ with the output $Y(t)$ is known to have the same form
- $$R_{xy}(\tau) = U(\tau) \cdot \tau \exp(-b\tau).$$
- Find auto correlation function of $Y(t)$ 4 M